Epistemic operations and formal epistemology.Contribution to the study of epistemic operations in scientific theories.

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as Chap. 4, in Mugur-Schächter, Mioara and van der Merwe, Alwyn (eds.), *Quantum mechanics, Mathematics, Cognition and Action. Proposals for a Formalized Epistemology*, Coll. « Fundamental Theories of Physics », Kluwer Academic Publishers, 2002, p. 37-71.

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ABSTRACT.

We want to investigate the various dimensions and constitutive problems of what a research designated as "formal epistemology" would be. The interest of such questions is related with the possibility or not of getting a privileged *point of view or axis of research* (that of the "formal"), that would afford the facts and problems tackled by precise (regional) epistemology of theories (for example, in physics) a better grasp at their wealth and variety ; it is hoped at the same time to hold the main structural lines organizing them according to some *comprehensive intelligibility*, more unifying and synthetic. Such a point of view would eventually allow a better mastership on the changes required in organizing the various knowledges, putting emphasis on their main directions, drawing up a rational inventory of them, and perhaps anticipating others.

We deal first with the "thought of changes" that any concern to the form cannot omit, as the meaning of forms is bound to contents, that are themselves submitted to *constructions* and *modifications*. We examine thenafter the notion of "epistemic operation" as instrument for creating new forms, at the theoretical and meta-theoretical levels. We subsequently analyze the characteristics of the *form* and of the *formal* and their relations to *contents* of knowledge, together with the notion of *object*, considering as well their dependence on the decision of a *subject* and on *conventional* choices. We finally inquire about the link between "epistemic operations" specified as above and algorithmic functions for knowledge statements, and we emphazise the risk of reductionism that a naturalistic conception of representation entails.

REDUCED ABSTRACT.

We ponder about the kind of problems and perspectives of a "formalized epistemology", by considering the advantages than one can get from a concern with the "*formal*", with its structural orientation, that would favour comprehensive, unifying and synthetic, intelligibility. We confront this perspective with that of the *changes* in knowledge, considering the relation between *form* and *meaning* for knowledge contents, and examine the notion of "*epistemic operation*" as instrumental for creating new forms, at the theoretical and meta-theoretical levels. Actually, the notions of *form*, of *formal* and of *object* are not independent of the problem of a *subject* that decides on *conventions* and *choices*. "Epistemic operations" might suggest a link with "algorithmic functions" for knowledge statements, that themselves entail the risk of reductionism in a naturalistic conception of representation.

KEY WORDS: Changes, Contents, Epistemology, Form, Meaning, Object, Operation.

1.

INTRODUCTION.

The expression "formal epistemology" covers a variety of possible definitions, and is widely open to interpretations[1]. One conceives it, intuitively, as oriented by a concern to theoretical forms, to their meaning and their scope, for a given area of knowledge or with regard to the relationships between different fields of knowledge as well. We actually intend here not so much to give a precise definition of "formal epistemology" than to focus on some problems that would be characteristic of this kind of investigation and to underline the latter's interest and limits. We therefore admit from the start some vagueness in the definition, in order to avoid limitation inside too narrow and artificial bounds : the question being, actually, to explore what it could be, resonably speaking.

Let us begin by circumscribing our project. It is not intended here to establish a kind of "morphology of the mind" such as that planned three quarters of a century ago by Ernst Cassirer in his beautiful work *The philosophy of symbolic forms*[2]. Cassirer extended in it his former inquiry on the "structure of the thought" as manifested "in mathematics and natural sciences", which he had expound in *Substance and function*[3] : he went on in the new three volumes book through the analysis of language, of mythical thought, and of the phenomenology of knowledge[4]. In any event, the idea that scientific thought is expressed through symbolic forms, as any other area of human thought, from its "infrastructure" to the "architectonic organization" of the superstructure that constitutes science[5], while it is aat the same time oriented towards the real world, this idea will remain, for us, a fundamental inspiration[6].

Questioning forms and formal problems, when one deals with precise sciences, leads in general to emphasize implicitly some current aspects of the

[1] The following reflexions have benefited from exchanges and discussions inside the working group entitled *Centre d'études pour la synthèse d'une épistémologie formelle* (CeSef, Paris), animated by Mioara Mugur-Schächter. I acknowledge the friends and colleagues of this group for the rich debates we had. A first version in french has already been published : Opérations épistémiques et épistémologie formelle. Contribution à l'étude des opérations épistémiques dans les théories scientifiques, *Principia* (Florianopolis, Br.), 3, 1999 (n°2, dezembro), -.
[2] Cassirer [1923-1929].
[3] Cassirer [1910].
[4] Subjects treated, respectively, in each of the three volumes of *The philosophy of symbolic forms.*
[5] Cassirer [1923-1929], vol. 3, p. 13-14.
[6] This idea has helped us in the orientation of other previous studies, in particular our *La matière dérobée* (*Stealing matter*), on the conceptions of contemporary physics (Paty [1988a]).

present scientific theories, with an endeavour to show how they are characteristic in this respect. We do not want, however, let ourselves be restricted to mere structural and static features, when our interest is directed, in particular from a comparative perspective, toward the *movement* by which these “formal” aspects are established, and is also impulsed by wondering how others could eventually be settled in the future. If we would restrict ourselves to examine only the formalizations of recent knowledges, we would be in danger to “lose the prey for the shade”, and to retain merely a “logically reconstructed” and schematic view of the sciences, to the detriment of their living reality and, so to speak, of the very substance or flesh of those knowledges, whose contents and forms, that were the fruit of elaborations, regularly undergo *modifying changes*. This “life of forms”, for the sciences as it is spoken of for the arts[7], must obviously be a constitutive dimension of any “formal epistemology”. We therefore feel important to make an inventory of this dimension, while maintaining in our approach, in the present contribution, a main concern to forms, to the “logic of forms”, to their analysis and their meanings, rather than to their historical circumstances, but without ignoring for all that the reality and the necessity of the changes which they emcompassed.

This is the reason why we shall begin this essay to “problematize” a “formal epistemology” with a reflection on the “thought of changes”. We shall examine thenafter the notion of “epistemic operation”, that shows itself, at both theoretical and meta-theoretical levels, as instrumental for the creation of new forms. Then we shall consider the features of the form and of the formal, and the relations they entertain with knowledge contents, and also the notion of object, considered as tributary to a subject's decision and to conventional choices. We shall conclude with questions about the link between “epistemic operations” thus specified and the idea of algorithmic function for knowledge statements, which will lead us to emphasize how a naturalistic conception of representation entails the risk of reductionism.

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EPISTEMOLOGY AND THOUGHT OF CHANGES

Einstein remarked that progress in physics leads to theoretical representations that are increasingly distant from the immediate form of our apprehension of reality. This is true also for other types of sciences and knowledges. Only, perhaps, disciplines whose mode of expression is narration - this concerns essentially history -, require a type of intelligibility that remains in a direct close contact with subjective impressions and immediate sensations, from which the facts of a reconstituted past are reactivated. (But even this does not

[7] See, for example, the classical work of Henti Focillon on the history of art, *La vie des formes* (Focillon [1981]). Actually, many recent or less recent books titles on esthetics and on the art call for the word “form”. The question of the relation between form and signification is obviously a central one in the domain of arts.

forbid a comprehension informed of more abstract and reflected elements, that go along with operations of judgement, with attributions of meaning).

Such a distance compels anybody who is still convinced that the elaborations of scientific knowledge aim to represent *reality* - reality of the natural world for natural sciences, social and human reality for the others, and it is still reality of the world[8] -, to take in consideration the increasingly numerous mediations laid between *representative thought* and its *object*. The successive states of these mediations can be traced back through history : it is possible to identify stages for them, considering each particular knowlege, enrooted in an overall culture and entertaining with the others contemporary knowledges relationships of mutual reciprocity. At each such stage, these knowledges and the culture in which they are embedded compose an organic piece of stuff in which types or norms of comprehension (or intelligibility) are determined. The question of the intelligibility of the world and of the nature of scientific knowledge is therefore inseparable from the consideration of their historical states. This consideration does not nevertheless imply any fundamental relativism that would deny or minimize the role and function of reason. One must admit as a truth about facts the existence of forms of knowledge and of justifications for reasoning that differ the ones from the others according to historical times and cultures. But this truth itself is liable of a rational, “scientific”, investigation.

This remark, whose implications about the nature of the scientific knowledge we examine elsewhere[9], aims only at underlining the importance and the extension of the question of the relationship between a new knowledge and the “tradition” in which it appears and, consequently, between future knowledge and present knowledge. Such a question stands at the basement of the considerations presented in what follows, and therefore widely overruns the case of present science as known to us, and to which we shall restrain ourselves here. It is notwithstanding fundamental to keep this dimension in mind, and to be aware that the scientific knowledge in which we are located is *not conatural* to us, but is the result of elaborations that are dependent on historical circumstances. Our attempts to give some purely formal and rational representation of it are themselves constructions, elaborations, that carry a part of contingency and of convention.

If the concern with formal and structural aspects, in which we are mainly interested here, has to omit, in a first stage, the consideration of historical circumstances, it nevertheless has to take as one of its starting facts the moving and evolutive nature of knowledge, that is to say the question of the changes affecting not only the objects of our representations, but also the modalities of these representations, up to our very manner to conceive them. Such changes affect equally, and possibly even more, the other areas of knowledge and of human experience that are not reducible to the scientific knowledge, such as aesthetics or moral, those other pillars of any culture, linked also, albeit differently, to the use of reason - and also to those other “classical” functions of thought that are imagination and memory.

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[8] In the case of mathematics it is also question of a reality, but it is an idea-like reality. One finds as well for them a larger distance taken from the intuitive forms of their origins, that were directly related to the sensorial experience.

[9] Paty [1999a].

In many areas of contemporary scientific knowledge, the ways in which one conceives what a phenomenon is, the object to which it refers or the explanation given to it, differ appreciably from the previous conceptions and practices, including those to which we ourselves have been educated. However, although we are conscious of these changes, since we experience them, we do not know yet how to interprete them, that is to say, as what and how do they modify our current manner of thinking and speaking[10]. This, indeed, would be *a priori* rather difficult to know by remaining inside that traditional thought. We are living these changes before knowing how to think them. It is however unavoidable for science, and it is essential for philosophy, to undertake somehow to think them. Is it possible, right now, to know something more, and to formulate the new thought required by the changes that we nowadays live, that are accomplished under our eyes ?

A first issue is to know what is to be hoped in this direction. The present situation is not entirely new : it has precedents, from which some lessons can be drawn. And firstly, that scientific thought is not isolated, that epistemic changes are only fully intelligible when considered inside a more general framework… And also that attention to these changes can be instructive without leading necessarily to a global view. Our ambition here must be kept modest, and still possibly fertile, otherwise generating mere illusions.

The modifications of representations uncovered by contemporary science are rich also of more positive teachings. We shall restrict ourselves here in evoking those of physics, but it would be possible to propose similar considerations for biology, geology, cosmology, mathematics, and social and human sciences as well. For instance, the theory of the relativity has shown the necessity to think anew and to rebuild notions that were considered the most obvious ones and the deepest anchored in our cognitive structures, such as space and time. The theory of the relativity (special and general) has also shown the necessity to re-examine what a physical theory is, as a symbolic, conceptual and formal construction, and to think more appropriately the particular role, in this construction, of mathematics - a science of pure forms that pertains notwithstanding to a real *world* of these forms[11].

Awareness of necessities of this kind is not radically new and has not appeared only with contemporary physics. We know, indeed, that the space-time continuum, that is the basis of the use of differential equations and stands as the foundation of all field theories, is an abstract entity elaborated by thought, and that its justification does not hold to some evidence of the intuition. The justification lays in part in the operative character of this built notion, that allows to describe and explain relevant phenomena. It lays, for another part, a fundamental and constitutive one, in the logic of a “mathematized” representation, first of space then of time, that guides our schemas of intelligibility, and that is itself not an *a priori* but the result of an elaboration process[12].

However, is is not actually a discovery that the space-time continuum is a mental, conceptual and symbolic construction, of which we are not assured

[10] Mugur-Schächter (1993, 1995, 1996).
[11] See Paty [1988a, 1993].
[12] Paty [1994c].

that the basic elements correspond to something truly real. It has been known since, at least, the science of *mechanics of material point and differential quantities*, based on magnitudes that, while being of physical scope, are idea-like ("idéelles"), and show as ideal abstractions of suppositious realities. This theory proved nevertheless extremely fertile through the efforts of physico-mathematicians and theoretical physicists, so as to conclude that it exceeds the schematic character of a mere ideality - mathematically convenient, if not always clear (as for such concepts as material point, fluxions and differentials), and far from *real* (in the sense of common realities). They did it on the basis of newtonian mechanics, with its general laws or principles, its relationships and its concepts, possibly reformulated, completed or generalized, and by extending it to complex solid bodies and to fluids, and also to gravitational attractions beteween more than two bodies. But they did it also by discussing - permanent concern of the epistemologist scientists[13] - the conditions of validity and the limits of application of these mathematical concepts in formulating physical magnitudes.

As for the changes occurred with quantum physics, these are also far from having been fully evaluated, at the level of the properly physical meaning of the theory as well as on the level of its cognitive impacts. Very early, physicists have been themselves preoccupied to generalize from it a methodology for physical theory and a philosophy of knowledge. But the worry to afford these new and remarkably fertile scientific perspectives a legitimacy that was in risk to be denied has fixed prematurely, and in a rigid way, the allowance of the interpretations. The conceptual difficulties raised in physical argumentation, in theoretical problematization and in epistemological analysis, were buried under the automatic replies of a shaped-on-measure philosophy. It was, indeed, necessary to reconsider, in the light of knowledges about the new phenomenal area, categories whose function had seemed to be definitively acquired, such as those of *causality* or *determinism*, and even those of *observation*, of *object* and of *objectivity*. But many others at the same time were omitted, and in particular concepts that could possibly be as much fundamental, such as those of *state* of a physical system and of *magnitude* characterizing such a state, and also the nature of the *probability* linked to the state. The quantum formalism gave mathematical definitions of concepts, but the "interpretation" was letting unclear their deep relationship with a direct physical meaning, by denying from the start any relevance to such a question, in the name of the alleged philosophical reasons[14].

The recent debates on the meaning of *non-local separability* for quantum systems[15] and the remaining doubts on the exact status and content of the "principle of reduction of the wave packet" (or *quantum problem of measurement*[16]) show remaining zones of obscurity in the question of the physical meaning of the quantum theory statements. This theory, in any case, is not restricted to quantum mechanics, and extends to quantum field theories which are today of a considerable importance, entailing in the long range implications at the

[13] From d'Alembert to Mach, Boltzmann, Einstein…
[14] Paty [1999d, h ; in press, a].
[15] Cf., f. inst., Bell [1987], Paty [1986], d'Espagnat [1994], Shimony [1993], Cohen, Horne & Stachel [1997a and b].
[16] Cf., in particular, Wheeler & Zurek [1983].

level of the conceptual meanings[17].

For this reason, when we shall evoke the general lessons to draw from quantum physics, we shall not mean the “philosophy of quantum mechanics” with the usual sense of this expression, that refers to Bohr's “complementarity” or to its variants according to others physicists, insofar as they put theoretical and conceptual criticism under the dependence of a philosophy of the observation. We shall mean, by these general lessons, all the theoretical, epistemological and philosophical questions raised by the *knowledge of quantum phenomena*, that constitutes one of the most important chapters of present physics[18].

Likewise, other contemporary physical theories have unusual implications on relationships that were usually considered as direct ones, but that actually require rigorous critical evaluations about the meaning of the concepts at stake. Consider, for example, the determinism-prevision close link, that is no longer relevant with the present theory of non-linear dynamical systems and “chaotic” phenomena. In such a case one should possibly distinguish between *prevision*, that deals with the description of a moving body running along its trajectory, by using space-time variables - concepts that loose here a great part of their fruitfulness -, and *prediction*, that is still provided indeed by the theory, but with respect to other variables - global ones, for example -, and to other concepts, such as that of “strange attractor” in the *theory of dynamical systems* [19]- that show as being characteristic of this kind of phenomena.

Epistemological re-evaluations therefore are still strongly needed if one wants to understand new knowledges according to the overall consistence of a wider intelligibility, such a deeper understanding going possibly along with reformulations. They obviously occur after these new knowledges have already been instaured, and they only come out from precise, “differential”, analyses of each of these knowledges considered as a specific one. One is then often surprised to see, in the transformations that have taken place, similar structural lines with respect to some characters, although they concern areas of knowledge that might be very different. One should consider, however, that such features show only in precise case studies, and that these rapprochements are not immediately visible to us. Many decantings are necessary before they impose themselves to the attention. And one generally inclines, in a first stage, to see in them mere coincidences, and pure analogy appears rightly as too weak an argument to lead to somewhat fundamental considerations[20].

One may wonder, however, whether a point of view or an axis for research would exist, from which it would have been possible to stand right from the start, and that would afford an overall perspective on the facts and problems to which the precise (i.e. “regional”) epistemology of these theories is confronted : a *perspective* that would be able to grasp together something of their wealth and variety, while giving at the same time an insight of the main structural lines that organize them according to some *overall intelligibility*, more unifying and synthetic. Getting at such a point of view might allow to better overcome the

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[17] Paty [1988a], chap. 8.
[18] Cf. in particular, Paty [1999c, d & h ; in press, a & b].
[19] Ruelle [1989, 1991].
[20] On analogy, see Paty [in press, d].

required changes in the organization of knowledge, to stress their main directions, to sketch a reasoned inventory of them, and perhaps to anticipate other ones.

Such an anticipation is not unthinkable, from the consideration of regularities or trends. One can, indeed, question the reason of such similarities, analogies and convergences, as observed in transformations occurred in different regions of knowledge, and to strive to bring them to some morphological feature that wrapped already these knowledge, as a kind of preset and predetermined conditioner : this latter, of which we were not conscious previously, was doomed to manifest itself at last. If such is the case, a previous examination, performed in a more systematic way, would have possibly been able to detect it without being in need to wait the contingent occurrence of the events. Such considerations would cling on a kind of meta-epistemology still to invent, if only it would be possible. They would be, for the epistemology of theoretical representations, a kind of an analogue of what would have been, to Hermann Minkowski's eyes, an priori mathematical theory of space-time if it had been developed before the physical theory of special relativity[21]. Let us already notice that hypotheses of this kind seem to be nourished by a wholly rational conception of the theories and knowledges under consideration and of their link to the meta-theories liable to frame them. It would still remain, anyhow, to specify what is to be understood by "rational conception", compatible with some allowance necessary for invention. Or, otherwise, would invention be merely only the "pragmatic" way of access to a progress of knowledge that would be fated in some absolute necessity ?

At all events, and without anticipately prejudging the replies to these questions, such considerations arouse the idea to bestow a particular attention, in these problems of readjustments and maybe also already at the elaborating phase, to what is set under the species of the "operative" and of the "formal". It is nevertheless still necessary to specify what is meant by these words, before trying to settle some marklands for a reflection on what one can possibly expect from it.

3.

EPISTEMIC OPERATIONS

We shall call "*epistemic operation*" an act of thought (or a series of such acts) through which a body of knowledge is constituted, be or not the nature of this act consciously perceived during the constitution of this knowledge. It can let itself be recognized as such in a further stage of the reflection, from the study of contents and procedures of knowledge in an given area.

A simple example of epistemic operation, taken from the methods of contemporary physics, is the *search for invariants* in establishing a physical theory : for example, a lagrangean that is Lorentz' invariant is suited to the conditions of special relativity ; or that is invariant under a given "gauge" symmetry operation considered as fundamental for some kind of dynamics of the

[21] Minkowski [1907, 1908].

interactions of “quantum” particles. This practice of physicists has become usual since the adoption of general relativity and of quantum physics, in the years 1920-1930. But its origin can be found in the mémoire on the dynamics of electron (“La dynamique de l'électron”) composed in 1905 by Henri Poincaré[22], when the latter planned to build a theory of gravitation modified with respect to that of Newton by imposing to it the condition of “covariance”, or invariance of the equations expressing laws under the “Lorentz' transformations” of space and time coordinates, and that means the submission to the principle of (special) relativity. This theoretical practice is related with the importance taken in physics since then by the notion of *group of transformation*, of *symmetry* or of *invariance*, subtended by the corresponding mathematical theory.

This importance has been fully understood with Einstein's theory of general relativity, and has received a first formal systematization with the theorem of (Emmy) Noether[23]. It afterwards has guided the elaboration of quantum mechanics and, later on, of the quantum field theory, up to the recent developments about the fundamental interaction fields obeying “gauge symmetry (or invariance)”.

By imposing itself, this “epistemic operation” has considerably modified the practice and the conception of physical theory. It is easy to formulate, once one knows it as justified ; but to account for its establishment is not liable of a simple scheme of explanation, unless distorting or ignoring historical reality, made of “facts that resist”. One cannot see it as a natural evidence that was required only because we conceive it so well afterwards (after its “invention”) without needing other justification. If it seems to be required in our retrospective look, it is because this latter is situated in a conceptual universe that took it as a reference, in a continuation of reorganizations of the knowledges and of the theoretical method of physics. But it is possible to trace back its origin inside the structural and conceptual changes occurred in physics at the beginning of XXth century. It was, in a first moment, *invented*, in a world of thought still marked by others conceptions and practices, through circumstances and for reasons that belong to historical study and, more precisely, to historical epistemology.

This example is suitable to make us see rather clearly, thanks to its relative simplicity, that we should distinguish *two levels* with respect to the object of our research : on the one hand, the level of *epistemic operations* in the *scientific work* properly speaking (here, the formulation of invariants in expressing laws and physical theories) and, on the other hand, the level of the same operations considered “to the second degree”, under the species of procedures that were concluded in stating the first ones, and whose formulation implies an historical aspect. This second level is that of the *constitution of epistemic operations*, of their elaboration, and it is hardly reducible to descriptions of simple operations. It has to do with the question of the formation of new ideas, of creation, in a general way, of “novelty” in science and, so to speak, with the question of the “emergence” of new forms in the cognitive thought.

One can consider also as *epistemic operations* the fact of reasoning in

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[22] Poincaré [1905b].
[23] Nœther [1918a & b].

the framework of some logical structures and of some categories of thought that inform our "interpretations", our manner to conceive the "meanings" of concepts and of theoretical statements. For example, concerning the propositions of a physical theory : an idea of *causality*, a specific understanding of *determinism*, the meaning ascribed to the concept of *probability*. Such ideas have repercussions as to the manner in which a problem is processed, this manner being possibly common to searchers and specialists in a given period, or likewise differing according to the individuals or the schools of thought. But they concern even more the way in which these knowledges are understood and justified. Moreover, this way is determinant as to the dynamics of the thought planning to go farther or not. For instance, the notion of "theoretical completeness", aroused by general relativity and by quantum physics, uncovers a program of research in view of a further physical theory, program that is qualified by the position adopted with respect to it[24]. None of the two mentioned theories is complete in the *strong sense* (of a *self-generation* of its objects), which is indeed that one of the present attempts at unifying field theories, and the question is actually to know whether they are complete *in a weaker sense* (are they sufficient for describing ascribed properties of their objects ?)[25]: such was, to Einstein, a fundamental condition required when endeavouring to go farther in a unifying description of physical objects.

Results obtained in a given science might challenge *epistemic operations* that would have otherwise been considered obvious. Such has been the case with the notion of causality that has been modified by the theory of special relativity which, by obliging to distinguish between the "space" and the "time" regions of the light cone[26], has entailed changes in our conception of the relationship between cause and effect. All the regions of the space-time diagram are not equivalent : if the "time region" is physical, the "space region" is non physical (there is no possible causality relationship between its hyperpoints). Henri Poincaré himself asserted it, with regard to his own ideas, concerning Minkowski's space-time[27].

On another hand, the notion of "probability amplitude" of quantum mechanics entails, if one thinks to it, a modification of the idea that we may frame of probabilities in physical theory. Probability is, without other specification, a mathematical notion. It is generally identified, when used in physics, to a frequency of events or, more exactly, to the limit of such a frequency, according to the law of great numbers. The construction of quantum theory makes use of it in an indirect way, through the "probability amplitude" that the wave function or the state vector is. Such a denomination reminds the physical meaning of a concept

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[24] Paty [1988b & in press, f].

[25] Such was the essence of the "EPR argument" (Einstein, Podolsky & Rosen [1935]) ; see Paty [1995 ; in press, e].

[26] The light cone, defined by the equation $x^2 + y^2 + z^2 - c^2t^2 = 0$, determines an inner "timelike region", such as $x^2 + y^2 + z^2 < c^2t^2$, and an external "space region", such as $x^2 + y^2 + z^2 > c^2t^2$. The former is the region of causal relationships between space-time points, the latter is that of acausal relationships ("non physical" region).

[27] Poincaré [1912]. Cf. Paty [1996]. This was shortly after Paul Langevin had discussed, in Poincaré's presence, in philosophical meetings, the physical implications of the new relativist conceptions on causality (Langevin [1911 a and b] ; cf. Paty [1996a, in press, f and g]).

that, in itself, is alien to the mathematical theory of probability and that, from the mathematical point of view, has the form of a vector of a Hilbert space. The “probability amplitude” provides theoretical probabilities, that are put afterward in relation with experimental frequencies of events.

But as the theory bears the meaning of its magnitudes (essentially through their relationships) before any effectuation of experiment (this latter providing only knowledge of their particular values), the probability obtained from the amplitude of probability possesses a theoretical meaning that is not reduced to that of the result of a given experiment, i.e. to the meaning of a frequency. It makes therefore sense to speak of the probability of an individual event with a theoretical and physical content, such as self-interference for one single photon, as Dirac anticipated it already in 1930[28], and this meaning differs from the probability of a cast of dice, that “is exhausted” by the dice being cast. I use here the verb “to exhaust” with a meaning that does prejudice on the independence of the probabilities of successive casts (“Un coup de dés jamais n'abolira le hasard”, “A cast of dice never will abolish chance”, Mallarmé). I mean that a single event concerning a classical magnitude is self-sufficient. On the contrary, measuring a singular quantum process through registering an event of a classical type lets unaffected, virtual-like, the spectral probability distribution of the magnitude under consideration. Both come into coïncidence, or better in correspondence, only when one gets interested in the results of mesurement for magnitudes conceived with classical conditions. Up to the very moment of the measure, the theoretical probability in the quantum sense (as given in the probability amplitude) carries all the information on the possible spectrum of the basis states, with the corresponding (numerical) probabilities. The possibility of the connection, and the result when it is performed, is sometimes expressed with the words “potentiality” or “propensity”[29] that are, actually, nothing more than intuitive substitutes for a conceptual jump : the jump existing between classical magnitudes endowed with numerical values and quantum magnitudes whose form is more complex[30].

We shall discuss later on the question to know whether epistemic operations are to be identified with *algorithms*. We shall have before to make clear what we mean by the word “formal” in the expression “formal epistemology” : we would like to show how this notion itself largely overflows that of algorithm. We will hinder equally to some extent on the notion of “object”, that allows to specify the purpose of a formal epistemology with respect to a general aspect of epistemic operations.

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[28] Dirac [1930]. See Paty [1999h ; in press, a].

[29] Heisenberg used to speak of “potentiality” and Popper of “propensity” (f. inst., Popper [1935]1968, [1982]). We let aside here the various interpretations that have been proposed for probability in this context, from Werner Heisenberg's “potentialities” to Karl Popper's “propensities” : note that these notions, as interesting as they might be to point at a problem, are purely “intuitive” and vague.

[30] Paty [in press, c, a & b].

TRYING TO DEFINE A "FORMAL EPISTEMOLOGY"

In the classical sense, *formal* opposes to *material*, as for example with Aristotle's formal cause (bearing on the idea or on the essence), or in the antagonistic couple of *form* and *matter*, or else in the most current meaning of the word "formalism" (the expression "purely formal" means : without real content, in the sense of a "material" one)[31]. This opposition, exploited by scholastics, refers to the *form* the relations existing between the elements of an operation of the understanding, whatever be the *matter*, or the *meaning*, of these last. *Form* with this acception is the source of an expression such as *formal relationship*, used for algebra and qualifying relationships that are valid for all numbers fingered by literal symbols[32]. It takes part also in a terminology used to designate laws - the *form of the law* -, and, in philosophy, for example in kantian philosophy, the laws of thought (pure forms of the sensible intuition or *a priori forms* of the sensibility - time and space -, *forms of the understanding* - categories -, *forms of reason* - ideas). In a different, more recent, sense aroused by *Gestalttheorie* - *theory of the form i*n psychology -, the form is what obliges to consider an element as part of an aggregate, of a totality, participating of its structure and of its laws.

By borrowing to this variety of acceptions, we come closer to a sense able to be directly used, bypassing the opposition between *form* and *content*, in an attempt to get at the fundamental characteristics of the gnoseological approach of knowledge contents. Gilles-G. Granger speaks of "formal contents" for mathematics, to distinguish them from the sciences of the empirical world, and at the same time to get them nearer : they are not empty forms, they also have *contents*, that can be attained at by their relationships as being forms that do not reduce to tautologies nor to mere logical expressions[33]. The *formal*, in this acception, is not identified to the purely logical which is, as for itself, empty of content. It does not oppose to content, and we shall have precisely to clarify the relationship of the formal and the content - some aspects of this will appear in the discussion of the notion of *object*.

Formal opposes to something of the kind of the empirical particular, to mere phenomenic description of objects, even if the latter's features were obtained by a theoretical approach in a natural science. *Mathematical physics* shows under a number of respects as a formal appoach, when compared indeed with experimental physics, but with *theoretical physics* as well, although it rather often converges to it, in such a way that both periodically come to an identification (as in analytical mechanics exposed by Joseph Louis Lagrange and Rowland Hamilton[34], or in Hermann Minkowski's theory - "formulation" - of special

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[31] Lalande [1926, 13th ed., 1980] : articles Cause, Formalisme, Forme, Formel.

[32] "Formal" calls for "actual" (as in the ancient and scholastic acception) and for "general" (as in algebra).

[33] Granger [1994, 1995].

[34] See Paty [1994b, 1999b].

relativty[35], or in Einstein's theorry of general relatvity, and in some presentations of quantum mechanics (notably those of Hermann Weyl and John von Neumann[36]), and also in many developments of gauge field theories (from the works of Yang and Mills, up to the most recent researches on quantum gravitation[37]). Mathematical physics deals with the *formal relations* between the mathematical quantities that supposedly concern physics. It is a formal of a mathematical nature, by opposition to the *physical content* which is primarily the worry of theoretical physics. But the opposition is only relative, and the noticed periodic identifications between mathematical physics and theoretical physics are nothing more than a indication that, as "formal" as they be with respect to nature, mathematical relationships are not alien to it, to the extent that they constitute the very form of the descriptions that theoretical physics gives of nature. In these privileged circumstances, the *form* succeeds in being the very expression of the *content*.

Taking again the example of invariants evoked previously, these last are caught by thought as formal relationships, but that express at the same time general and fundamental properties of physical system and of the magnitudes describing them in the physical theory. Far from being external and superficial, the *formal relationship* is therefore capable to express *contents*, bearers of meanings : it is a privileged way, maybe sometimes the only way, to express these meanings. In this manner Poincaré considered "mathematical analogies"[38], and Einstein "formal analogies"[39], which happen to be equivalent (mathematics are the formal for the physicist). Poincaré and, some time later, Einstein did not hesitate in speaking of a "heuristic of the mathematical formalism", that drifts physical thought, precisely because this formalism, in the cases which they considered, was informed of, and even impregnated with, the physical meanings that it served to express[40].

Although the nature and the role of the formalism in mathematical physics advises us somehow that attention to the formal is not exclusive of contents and meanings, this does not tell us yet what one should conceive as being the purpose of a "formal epistemology".

Does one mean to study the formal aspects of epistemology as, for example, other people study under their formal aspects such or such realizations in an area of artistic expression ? Of that kind, the literary critical analysis bearing on the form for the poetry of Baudelaire or of Mallarmé. But one can also conceive a concern for the form that does not aim only to describe or to characterize existing "styles" of work or thought[41], but whose worry is to find new forms of expression, or new manners to generate new contents - perhaps new meanings -, through a privileged attention to the form, for example by imposing *formal constraints* to achieve a work. This tendency is frequent in contemporary art, in painting, in music or in literature. The worry might also be to practice work

[35] Minkowski [1907, 1908] ; cf. Walter [1996].
[36] Weyl [1928], von Neumann [1932].
[37] Notably, Ashtekhar [1989]. Cf. Kouneiher [1998].
[38] Poincaré [1897]. Cf. Paty [in press, d].
[39] Einstein [1912]. Cf. Paty [1993], chap. 4, p. 164-172
[40] Paty [1993], chap. 5.
[41] On the notion of *style* in science, see Granger [1968], Paty [1990], chap. 4, [1993a], chap. 1.

on the form in order to generate original expressions, such as the exercises of the *Oulipo* group[42], submitting poetical or literary writing to formal constraints[43] that produce aesthetic innovations and effects of meaning.

The *epistemological reflection* deals partly about forms, and takes itself forms that depend on the modalities of its approach and that are connected, in a somehow more direct and compelling manner than aesthetical forms, to questions of meaning : a meaning existing beforehand, but non necessarily already given, and which is, precisely, to be put under day light from beneath facts and appearances. This reflection is not, for sure, free creation of form ; as a critical study by the thought of an "object" existing outside of it, i.e. as scientific knowledge, it shares with this last the external existence of its object, and bears constraints that hold to this exteriority. Then, precisely, it is to the exteriority of the object of scientific knowledge - including mathematics - that one may rightly refer essentially the formal constraints of its representations. Is it possible to have a similar situation, to some degree, with epistemology as a reflexive knowledge of science ? We shall let aside, for the moment, the question of formal constraint possibly imposed also by the gnoseological structures of the knowing subject.

A "formal epistemology" has, so to speak, to be in connivance with its object, and this excludes any game on formalisms that would be alien to this object, and it excludes as well any identification with a formalism that would not be directly relevant to it (this raising already the question of algorithms and of exercising one's thought with arbitrary models). It can focus on studying *forms* (and *formalisms*) that characterize scientific knowledge, and the operations as well by which these forms are established (these operations we called *epistemic*). This being considered, the margin still remains large to define the manner, or manners, to practice it, which incidentally is an advantage : it remains *a priori* possible to choose the manner that seems to us the most fruitful, the most adequate and satisfactory with respect to the object in view, i.e. to the foreseen purpose.

On another hand, *formal* is not to be identified with *quantified*, as we are informed by the very mathematization of physical magnitudes, that invites us to distinguish, among the consequences of this mathematization, the *qualitative* - understood in the modern sense, and not in that of Aristotle and of the scolastics -, in the *disposition* and the *order of magnitude*, reflecting the conceptual *idea*, and the *quantity*, whose informations are of another kind[44]. And again, *formalization* does not mean schematization - similarly, to describe theoretically the form of an animal is not to reduce it to its skeleton -, but it concerns the fact and the manner of *taking a form*. The question of the *form* under which scientific knowledge and the particular sciences show in a *factual* way, as constituted and as being in a constituting process, let see that form has to do with *intelligibility*.

What is the intelligibility given by a given scientific knowledge with

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[42] *Ouvroir de littérature potentielle* (*Opening device of potential litterature)*, created in 1960, animated notably by Italo Calvino, François Le Lionnais, Georges Pérec, Raymond Queneau, Jacques Roubaud (Oulipo [1977]).

[43] See, for inst., *La disparition*, by Georges Pérec (Pérec [1969]), a novel written without using the letter *e*, a vowel yet so much frequent….

[44] See Paty [1994a].

the form it takes ? Furthermore, if a narrow link exists between form and structure, it appears equally wishable to distinguish the two, *form* being more untied, elastic and manageable than *structures*[45]. The form expresses the structure, but only partly : it also expresses others characters, compatible with the structure. On the other hand, a same piece of knowledge may dress itself with various forms, and one might wonder whether there corresponds different intelligibilities for these forms. This question, linked to that of "interpretations", appears actually as a fundamental one : it is possible to see there one of the key-articulations of a "formal epistemology".

"Epistemology of formal knowledge", "epistemology of form", "formal epistemology"… This metascientific approach of the function of form in knowledge can be seen as a methodology by its heuristic-oriented choice. It remains to know to what extent it can pretend to generate norms of reasoning, for example of conceptualization, allowing to reproduce or to anticipate *inventions* (that are precisely *inventions of forms*). That is not evident *a priori* considering what we know up to now about understanding and creating concepts. To reconsider a concept is, in some way, to recreate it, and this is produced in the unit of a singular, subjective, thought - considering here subjectivity in the epistemic sense, i. e. as being the center of the acts of reason.

These reasonings creating new forms, do not only operate rules, numerable and which could be classified according to some typology, but are inscribed inside a consciousness that mobilizes, in the acts of the understanding, many other instances than only the identified elements of the questions to solve. By these other instances, we do not mean psychology or sociology, of which it is needed to be freed when considering the formal that worries us here - such a rule must be obeyed also in epistemology when one deals with the scientific contents of concept and theories -, but we mean factors that are not made explicit and are generally let unconscious, which play likewise their part in the economy of the acts of reason[46].

We are led hencefore to situate *formal epistemology* and *epistemic operations* with respect to the *judgements* and to the *decisions* (or to the *choices*) set by the subject of knowledge, insofar as it is an *epistemic subject,* and therefore to specify what remains of the subject - unique center of intelligibility - for an "objective" knowledge that wants to abstract, as a matter of principle, the singularity of the subjective by requiring only *operation*, *process* and *content*, "without a subject". Although knowledge is established, evaluated, communicated, through *acts of creation* and of *judgement*, a subject *as the center of these acts* is undeniably required by an epistemology, even a formal one. Without it, knowledge would bear on *contents without the intelligibility* or, at best, with an anonymous and abstract "intelligibility" : but one is in right to wonder whether the very notion of *content* would then be only conceivable. Could a knowledge content be merely schematic ?

[45] On *structure* in science, see for ex., Stegmuller [1973, 1979]. On structure for history, cf. Foucault [1969], Veyne [1971, 1978], and, for anthropological representations, Levi-Strauss [1958].

[46] On scientific invention, see Hadamard [1945], and, on the rationality of this invention in several scientists, Paty [1999e].

The presence of a subject in filigree appears under some elements that, upstream or downstream of any knowledge, qualify the conditions, the effects or the modalities of these acts. We only mention them here, and their study belongs to epistemology in the general sense, structural or historical : conditions of possibility[47], fields of rationality, styles, programs, intelligibility, intuition…[48] The difference, here, between formal epistemology and epistemology in the general sense is that, if the second takes these elements as objects for study, the first one will only take them into account as something given and conditioning that it pretends to transcend or, more exactly, with respect to which it situates by searching "structural invariants".

5.

OBJECT AND CONVENTION

Examining epistemic operations leads, as epistemology in the current sense does it also on its own side, to disclose in the operations of knowledge a preliminary part of the organization by the understanding, that "prepares" the object of knowledge, or rather the conditions of its identification[49].

All sciences, be them exact, natural or social ones, are to-day conscious of the necessity to *criticize the notion of object*, especially by taking into account that this *object* is defined by a separation from the subject that puts it : the critique of the notion of object entails, correlatively, that of the notion of *objectivity*. On the one hand, no object can be designated without a *mental act* or an *intention of the subject*. On the other hand, any object is defined by its *distinction* or its separation *from a background* against which it stands out. It is known also that one must not restrict science, as for the conceptualisation of its objects, to characters afforded by *common sense* : here again, quantum physics has allowed to benefit some particularly precise lessons, to which we refer without being able to detail them here[50].

Many have often interpreted as a "de-ontologization" of the sciences the transformations *from objects to relationships* that have been characterized above all in *mathematics*, especially since they abandoned a unique idea of geometry ; it has been diagnosticated in *physics* as well due to the mathematization of physical theory, and all the more so as physical theory takes increasingly the form of a mathematical physics. But the notion of *object* is not for all that abolished in that of mere *relationship*. To deal, indeed, with relationships

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[47] Kant [1781-1987].
[48] Granger [1968], Lakatos [1970], Zahar [1989], chap. 1, Paty [1988, 1990, 1993], …
[49] The word "preparation", rightly pointed at by M. Mugur-Schächter (*op. cit.*), comes from quantum mechanics. But consciousness of this kind of procedure (which is a somewhat universal one) was also present in epistemological reflections previous to it ; quantum mechanics has contributed in giving it a pecular flavour and a better precision (see, in particular, Margenau (1978)).
[50] Paty [1988, 1986, 1999d].

is to deal with elements interplaying trough these relationships : in order to relate, *something* is needed - say : *elements* - that be *related* together, even if the nature of such “elements” is problematical or relative. At all events are they the “objects” of the relationship, and it is in this way that one tends henceforth to conceive the very notion of object, in mathematics, and in physics as well.

It is true, however, as the movement of setting relationships is progressing with increasing degree of formalization and of abstraction of theories, that the proper *elements* or objects of the relationships transform themselves, in their turn, in other *relationships*, at least partly. *Elements* might be, and in a “complete” physical or mathematical theory (even in the weak sense evoked earlier), actually are, *given in and by the relationship* itself (such is eminently the case with quantum theory). But, to the extent that the system of these relationships is not transparent or tautological, it expresses a structured content that resists dissolution into mere relational (we ought here to specify what is meant by that : for example, simply *relations* without *things* being related). This content that resists and that to be known needs to be explored, has therefore an attribute that is generally referred to as “reality”. An example as simple and elementary (at least at first inspection) as the system of the integer numbers of arithmetics can directly illustrate that complexity of relations, that “reality” of some kind that subtends them. Even by generating the totality of integers by being given the *number 1* as unit and the operation of *addition* of that unit to itself, and the cumulative *repetition* of this operation, one can not know in advance all their properties, for example which of them are prime numbers and their relationships among themselves (such as Fermat's “great theorem”). Notice however that the *repetition of elementary additions* in the generating relationship implies that one processes not only numbers but numbers of operations on these numbers…, and that the mode of generation was simple only apparently. (Notice also that *relationship* is inherent to the mode of *generation* of numbers, and therefore to the “essence” of these latter).

Physics provides good examples as well. Take the concept of field, once cleared by the special theory of relativity of the material or objectal support of ether : the *field* is defined physically only through the *equations of the field*. Or again, the indistinguishability of identical quantum systems or “particles”, that defines *at the same time the relationship and the object of this relationship*, and the symmetries of elementary particle that determine these latter in their relationships between them through their fields of interaction (gauge fields)[51]..

It might be more correct to say that it is this very *relational* that has become “*as concrete as something real*”. If the transformation, that is always possible, of elements into others, more relational ones, makes their “objectal” nature a more relative one, it remains that their narrow relationship between them appears finally as having all the characters of what one conceives as an object, endowed with a proper consistency. A nucleus of structured relationships closely woven does not leave anything to wish for the notion of object, once this one is freed from the substances of ancient metaphysical doctrines. Hence the object, be it mathematical or physical (and, by extension, chemical, biological…), loses its

[51] Paty [1999c, h ; in press, a].

traditional relation with a directly ontological perspective. The category of "being" respectively to it is, if not deleted, at least distanciated from it through relational mediations. But predicates of existence, as for them, remain, as their criteria are of an epistemological nature.

The ambiguities of the word "ontology" do not justify therefore to give up the notion of "object". Furthermore, a representation or a theory is always representation or theory *of something*~: by definition, this "something", whose necessity here is a logical one, is the "object", that this theory designates. By having already oriented the representation or the theory at stake, one has qualified this object in some way. It is clearly through an *act of the thought* that we have designated it so as to be described by the representation or the theory. In such an act stands a *choice*, that separates this object from the "rest" and, indicating a program, comprises a part of convention, that the object therefore carries with it. The conception of *objectivity* is correlatively affected by this : it is not only *given* (as thought is drifted along by the object), but also *decided* (according to some norms or conditions) and *built* as well (in relation with our choices to settle the object).

The chosen convention depends on the concepts and on the theoretical system interweaving them in order to describe the object. It is relative to this system, and alternative conventions are thinkable, that do not bear only on the theory, but on the totality of the elements of meaning for the theory that are in keeping with a definite intelligibility, and whose criteria are themselves partly metatheoretical. It urges to study precisely the interplay of these two notions, *object* and *convention*, that call for requirements of various kinds related to the types of representative theories that one might consider.

We shall restrain on this to only a few more words. One often gives credit to quantum mechanics of having eliminated the notion of object, at all events insofar as this last would be thought independently of any act of observation and of conceptualization and would preexist to such acts. For sure - and it is actually commonly admitted, as previously stated, well beyond quantum physics - the object as such has been separated from the rest (or the conditions of its separation have been prepared) through an operation of the mind. But this "condition of possibility" for thinking the object does not exhaust the description one can perform : actually, it only makes it possible by opening the field for its realization.

One might consider, in this respect, that the specific problem of *quantum mechanics* is that of the nature of the acts - of thought and of operations - that are required to get access to the description of the object, or, more exactly, of its *state*. I have tried elsewhere - and here I cannot stretch myself out - to show, from this standpoint, that quantum systems and their states can be conceived in terms of *objects* endowed with properties, at the cost of an extension of meaning, already realized in the practical understanding of physicists, but not yet explicitly admitted, of the notions of *physical magnitude* and of *state function* for a system described by such magnitudes, beyond their usually accepted meaning of *direct numerically valued quantity*[52]. Notice again only that one could have find,

[52] Paty [1999h ; in press, a, c].

upstream quantum physics, circumstances where "objects", represented by abstract magnitudes, had already lost qualities habitually attributed by common sense (such was already the light wave, spreading in all space, or the field without the support of an ether, as evoked above). Let us remark also that the concept of state function, that entered already Hamilton's mechanics, was suggested to him by optics with the application of a principle of minimization - another epistemic operation that refers to a more ancient origin -, in mechanics as well as in optics, namely the principle of least action.

Let us retain the following : one has known how to build a representation that can be said to correspond to a kind of object, without knowing for all that its deep justification - except that it is operative. (I mean that it suffices to anything that is needed to describe the object and the phenomena related to it.) We nevertheless know that the possible epistemological difficulties stand rather at the level of the - conceptual and theoretical - *tool*, than at the level of the nature of the *object* whose theory we are dealing with. In fact, the tool has been adjusted according to the object it was aimed at : not directly, but through the construction and the mediation of a theory adequate to represent it.

The *operative tool* that the quantum rule, or algorithm, is, has been elaborated through its adaptation to the necessity to represent a coherent world of objects - the world of quantum objects - liable to give account of quantum phenomena. The tool and the elements of the representation are - by the logic of their making - made of the same stuff. And the formalism of the state functions defined in Hilbert spaces and of operators acting on them, that aim to represent quantum systems, is accompanied by its rule, by construction. But at their level, in the interplay of relations of their world, the "objects" that these state functions designate - namely, the quantum systems - do not need, in order to be thought, to be referred at any time to the tool that, after having constructed them, detects them, that is to say to the rule, in terms of statistical probabilities and of reduction. They are actually thought accordingly to their designation, that is to say to the theoretical formalism itself[53]. In a way, the question of the nature of the tool is not so much that of the represented quantum object, than that of the relation between the quantum representation and the classical representation adapted to the experimental devices[54].

These remarks suggest a movement so to speak opposite to the progressive "syntaxization of semantics" which expressed, in the terms of the philosophy of language, the mathematization of physical contents, or the transformation of the object into the relation[55]. If one considers the effective evolution of the question of the interpretation of quantum mechanics since the first debates, and the subsequent familiarization acquired in this area by physicists, up to the recent knowledges and to the reinterpretations that may be consequently formulated, it is tempting to speak of a *semantization of the syntax*. The description of quantum physical systems was conceived previously only by means of operations, while it can be given henceforth, accordingly with what we have just suggested, in terms of physical systems, conceived as objects having

[53] It is possible in such a way to "think quantum non-separability" (Paty [1986]).
[54] Paty [in press, b].
[55] The expression is from Ernst Cassirer, cf. Cassirer [1910]. See Granger [1994].

properties (indeed, at the cost of transformations in our definitions of the physical magnitudes used to describe such properties).

The previous "syntax" remained external to the physical content properly speaking, since it was considered as restricted to purely formal means of the description, without coming to a decision on the physical existence of these systems. A full achievement of the *program of the semantization of the theory* would be to formulate quantum theory as the theory of a category of physical objects and of their physical properties. This would be a necessary task before going again to new properly physical syntaxizations. (One should be able to substitute the axiomatic formulation *à la* von Neumann[56], expressed in terms of utilization rules of the formalism, by an new equivalent axiomatic formulation in terms of physical properties concerning the quantum level).

6.

ALGORITHM AND NATURALISTIC REDUCTION OF REPRESENTATION

The notion of *epistemic operation* puts naturally the problem to know in what measure such operations can be algorithmic ones. One can conceive them as such when these are simple operations in the scientific work, such as the case, mentioned earlier, of a search of invariants, for example. Albeit, if one can conceive algorithmically an already known invariant, it is not obvious to conceive an algorithm for a search of invariants. For complex operations bearing on the acquisition of more precise knowledge, the answer is much more difficult, and on this possibility opinions diverge. The experts in artificial intelligence, and the adepts of it, will reply willingly by the affirmative. In the long range, according to them, all cognitive operations, including scientific inventions, will be able to be reconstituted : they are already proposing many models for it and for relatively simple situations, proclaiming its obvious necessity of principle in all cases, and sending the skeptics back to the old archaic dualism of matter and mind[57]. Is the brain, so do they claim, different from a machine (a neuronal one, indeed) ?

One may, actually, estimate that this question, by some aspects, is not unprecedented. The cartesian research of a method to get a secured knowledge, or that, leibnizian one, of a universal characteristic allowing to formulate in a unique and perfectly logical language the totality of knowledge, and even the *parti-pris* of demonstration *more geometrico* of the statements in Spinoza's *Ethics*, not to go back to Aristotle, testify the permanence of a somewhat similar concern through the history of philosophy; at least as for the possibility of a powerful algorithm liable of founding, gathering and organizing true knowledges. This wish for synthesis by means of a formal unity was not necessarily a reductionist one : it was protected, in Descartes, by the dualism of the acted matter and the thinking

[56] Neumann [1932].
[57] Changeux [1983], Damasio [1994].

mind, and the spinozian monism used to let all the room for the specificities of the various sciences. The tendency appears to be a quite different one with the modern idea of the brain as a machine : the stake here is, actually, not only that one of an algorithm for the representation, that would remain inner to it; but its naturalistic reduction[58] - a question on which we cannot extrend ourselves in this contribution.

The claim of algorithms for the scientific knowledge has had the opportunity to be strengthened in our times by the logicist ideas of the philosophers of the Vienna and Berlin circles[59] and of their successors, dissenters or not, up to analytic philosophy[60]. For the logical positivists and empiricists, science would generate a philosophy of knowledge that would be compelling (the so called "scientific philosophy", presented and supported by Hans Reichenbach[61]), a philosophy enrooted in the data of experience taken as the fundamental reference for knowledge. The search, with Rudolf Carnap, for an inductive logic, be it only a probabilistic one[62], was subtended by the idea that any scientific knowledge can be reduced to a rule, valid everywhere, perhaps in all times, that would have to be discovered : and this is also to think of scientific knowledge on an algorithmic mode. Asserting confidence in methodology[63], and claiming the normative legitimacy of a "rational reconstruction" of the scientific knowledge (Reichenbach, Popper[64], among others) that allows to correct the irrationalities due to the intervention of the subject in a knowledge whose vocation is to be an objective one, i.e. a knowledge "without subject", these positions go equally in the same direction.

One may also consider as an indication or an effect of this view the popperian "third world" of the forms of objective knowledge[65] : this impersonal universe of ideas is supposed to be that of pure rationality, cleared of all affects and of chance, and even of materiality (sent back respectively to the second world and to the first one). This world of created forms, without the acts of creation, is akin to the reservation or to the museum : a "museum of ideas", "virtual" before the recent common use of the word, where one draws to formulate other ideas, and conceivable only related to the two other worlds. This idea harbors, it seems, a rather platonician one of purifying the world of ideas from perishable elements such as matter, flesh, affects and feelings, that make the individual subject, while protecting a logic of the reconstruction that is not very different from the algorithmic function.

This being said, one will acknowledge that an algorithm, with a logical application, is not to be identified *per ipse* to a machine, for it is not

[58] See the quite interesting dialogue between Jean-Pierre Changeux and Paul Ricœur (Changeux & Ricœur [1998]).
[59] Wiener Kreis [1929], Hahn, Neurath & Carnap [1929], Soulez [1986].
[60] See Joëlle Proust's book on the "questions of form", of logic and of analytical statements, from Kant to Carnap (Proust [1986])
[61] Reichenbach [1951, 1959].
[62] Carnap [1935, 1950], Jeffrey [1980].
[63] Bunge [1983]. Alberto Cupani (Cupani [1998]) rightly reminds us, however, that, for Bunge, the method is not a "recipe" that would be mechanically or automatically applied.
[64] Reichenbach [1938, 1951, 1959, 1978], Popper [1935, 1972].
[65] Popper [1972], p. 154.

necessarily overlaid with the idea of reproducing the totality of operations of knowledge. One has also to take into account a widening of the notion of machine, that would include the possibility for new forms or *emergent* properties to appear, concerning material systems as well as the novelty sprang out in the space of ideas. But invoking a powerful algorithm or a machine, even with organic properties, as being to reproduce or to describe the process of acquisition of a fundamentally new representation gives rise to reserves, without any need to invoke dualism and holding on the contrary an ontologically monist position : these reserves are similar to those one can soundly oppose to reductionism and to a naturalistic conception of knowledge and of values. It is possible to emit them without denying for all that any interest to “epistemic operations”.

It is possible to conceive an epistemic operation generating an algorithm, such as that of the invariants considered above or - another example - the leibnizian differential calculus, and, once the algorithm has been invented, to reorganize or to reconstitute with its help all the ascertained or predictable properties of a representation in a chosen referential of meaning. The algorithm can demonstrate its fruitfulness in the resolution of many problems, and even perhaps contribute to pose new problems and to solve them. But can it make rise, by itself, a qualitatively new property or knowledge ? It appears logically possible only if the algorithm contains in itself this break relative to its antecedents that makes the new. But would not we have quitted, ever since then, the demarcated framework of the “epistemic operations” that can be formulated ?

A machine to produce some “conceptually new” remains, so far as we know, still to be invented. This considered, should we think of such a possibility as being absolutely unthinkable for the future ? The question, to which “artificial intelligence” would like to give a positive answer, depends on what one can call “qualitatively new” : this characteristic escapes, at any event, the content properly speaking - i.e. internal -, of a knowledge, referred to its framework of thought and situated in the universe of meanings. It is difficult to imagine the existence, or even only the possibility, of a “machine to produce sense”, in the common acception of these terms, at all events in the absence of a thought that would be at the origin of this sense or that could “read” it.

Evaluating concepts and their possible character of novelty belongs to signification, and until further notice, it is the human thought, fruit of a brain inseparable from a body and from a practice of life, and setting aims to itself, by will or by desires, that imposes its meanings on the machine. A “machine to produce sense” would have to possess such properties, and for sure still others, including feelings and psychology : such a machine would then astonishingly look like man in society, whose outgrowth by nature has resulted from a very long - and maybe improbable - history and whose origin has been lost in the night of times : the history of maturations, renewals, transmissions and exchanges, resulting from the diversification of the human phylum, biological, social and cultural, and from the chance of accidents.

As a consequence, the fundamental question is reduced to the following one : is it possible to think of an algorithm capable to generate, for knowledge, a meaning that would differ from those that are available to us but that could appear to us as legitimate, and perhaps more certain as these ones ? It seems that one enters, with this type of problems, in an unending chain of implications and in an infinite multiplicity of open ways that the machine will have much harm to solve.

On the contrary, human thought, that has stemmed from matter, indeed, does not calculate all the possibilities as machine does, but cuts across the offered combinations and makes its choices well before having exhausted all of them[66]. It puts, well simply, the meaning that shows to it - according to its judgment, that is possibly subjective only in some sense - the sudden clearness of an intelligibility. That one, be it cartesian evidence, spinozian knowledge of the third kind, illumination of the intuition as for Poincaré, Einstein and other contemporary thinkers, seems doomed to durably escape reductionist description. It is so because this intelligibility must call, if it wants to get foundations, to other ones in an endless regressive chain, as with the pascalian considerations on the situation of man's intelligence in the world, leaning on reasons that he believes he understands by experience but that, when he questions them, reveal themselves to be a bottomless well[67].

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